Preface: Symmetries and Integrability of Difference Equations

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PREFACE

Symmetries and Integrability of Difference Equations

Special issue dedicated to the subject of the SIDE IV meeting, held in Tokyo, 27 November–1 December 2000

Background

For well over a decade now, there has been growing interest in the integrability of discrete systems, i.e., systems that can be described by ordinary or partial difference equations allowing exact methods of solution. Such systems turn up in a wide variety of areas and have an extensive range of applications: mathematical physics, numerical analysis, computer science, mathematical biology, economics, combinatorics, statistical physics, the theory of special functions, asymptotic analysis, discrete geometry, mathematical design, quantum physics, quantum field theory, and so on.

The theory of partial and ordinary difference equations has historically been less well developed than the analogous theories for differential equations and there is an urgency to correct this imbalance. Discrete systems have recently played an important role in the theory of dynamical systems, but their study has often been restricted to the context of chaos, fractals and disordered systems. The development of rigorous analytic tools for dealing with difference equations is still in its infancy; what is still greatly needed is further study on, for example, their symmetries and symmetry reductions, asymptotology and singularity analysis, together with the problems of classification of difference equations. The situation has improved in recent years due to a number of remarkable developments in the theory of discrete systems, and this has affected a broad range of fields, including numerical analysis, cellular automata, discrete geometry, commuting and multi-valued mappings, discrete Painlevé equations, quantum mappings, multi-dimensional structures, similarity reductions, q-special functions, symplectic maps, representations of quantum groups, symmetries of difference equations, etc.

We believe that the domain of discrete systems forms nowadays one of the focal points in research in integrable systems. Also, beyond the study of integrability itself, there are numerous applications to disciplines where finite-difference equations play a role in, for example, computation and computer science, complexity, mathematical biology, dynamical systems theory, and many other fields.

SIDE meetings

The purpose of the international SIDE meetings on *Symmetries and Integrability of Difference Equations* is to provide an interdisciplinary platform where researchers coming from quite different backgrounds and working in different disciplines, all of whom share an interest in the methods, structures and techniques associated with integrable discrete systems in one way or another, can meet and interact. These meetings have been successful in pulling together workers from a diverse variety of fields, including classical and quantum physics, computer science, mathematical biology, economics, numerical analysis, discrete geometry, etc.

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The SIDE meetings started in 1994, when the first meeting of this type was held in Esterel, Quebec (near Montreal, Canada). Subsequent meetings have taken place biannually, in 1996 at the University of Kent in Canterbury (UK) and in 1998 at Sabaudia (Italy).

The SIDE IV meeting was held at the University of Tokyo from 27 November to 1 December, 2000. The scientific committee was chaired by R Hirota (honorary chairperson) and K Okamoto (chairperson) while the organizing committee was chaired by M Toda (honorary chairperson) and J Satsuma (chairperson). The local organization was in the hands of J Satsuma and T Tokihiro.

Proceedings of all the previous SIDE meetings have been published, the 1994 and 1998 meetings (edited respectively by D Levi, L Vinet and P Winternitz, and by D Levi and O Ragnisco) as volumes in the *CRM Proceedings and Lecture Notes* series (AMS Publications), and the 1996 meeting (edited by P Clarkson and F W Nijhoff) as Volume 255 in the *LMS Lecture Notes* series.

This current special issue of *Journal of Physics A: Mathematical and General* is dedicated to the subject of the SIDE IV meeting in Tokyo. Most of the contributors to this issue took part in the meeting, but this volume does not aim to be a proceedings in the usual sense of the word: the contributions do not necessarily coincide with the reports presented at the meeting, nor are the contributors restricted exclusively to those people that were present. The intention of the special issue is to benefit from the occasion offered by the SIDE IV meeting to highlight the important new areas of discrete integrability, by collecting together in one single volume a selection of articles reflecting the scope of the meeting. All contributions to this special issue are original research papers, but by collecting them together we feel that we offer a better context for the work and an insight into the new directions where this research is leading.

Subdivision of the special issue into topics

The binding theme of all the contributions is that of discrete systems and integrability, but the spread of topics covered in this special issue is wide. We have tried to identify the main directions of research in the subject by grouping papers together according to topics. Thus, we have arrived at the following subdivision of this special issue:

Symmetries and conservation laws. Here we have six papers highlighting various aspects of symmetries of difference equations. The paper by P E Hydon shows how to construct conservation laws for partial difference equations from a general perspective; generalized symmetries of difference–difference equations of Toda type are considered in the paper by D Levi and L Martina; the paper by I T Habibullin and T G Kazakova links the issue of symmetries for integrable chains to that of boundary conditions, while the paper by R Hirota *et al* presents a method of finding conserved quantities of nonlinear ordinary difference equations. The papers by C Budd and V Dorodnitsyn and by A K Common and M Musette investigate the symmetries of more specific equations and the discretization of special solutions.

Geometry and discrete systems. Under this heading we have the papers by M Mañas and by A Doliwa *et al* studying aspects of quadrilateral lattices: the former looks at fundamental transformations, whereas the latter focuses on (integrable) reductions. Furthermore, we have in this section the paper by M Kamata and A Nakamula presenting a q-analogue of the ADHMN construction of Yang–Mills instantons, and the paper by V Adler, presenting ways of formulating integrable discrete equations on arbitrary planar graphs.

Discrete systems and quantum mechanics. In this section we have four papers. The article by R K Bullough *et al* deals with q-deformed quantum lattices; the article by C Chryssomalakos and A Turbiner studies mappings preserving quantum commutation relations. F H Szafraniec discusses in his paper the duality between finite difference operators and creation and annihilation operators. The paper by S Sergeev introduces quantum mappings on a multidimensional lattice leading to (2+1)-dimensional quantized evolution systems.

Affine Weyl groups and (discrete) Painlevé equations. In this section we have three papers: the paper by R Conte and M Musette, presenting a new derivation of the birational transformation (sometimes referred to as Bäcklund–Schlesinger transformations) associated with the Painlevé VI equation; the paper by Y Ohta *et al* embedding a large parameter-family of discrete Painlevé equations in a multidimensional lattice representing the actions of the affine Weyl group $E_8^{(1)}$; and the paper by T Takenawa, who connects the notion of algebraic entropy to the structure of the space of initial values for birational dynamical maps through the investigation of the action of the relevant Picard group.

KP lattices and differential–difference hierarchies. A number of papers are related to the famous discrete KP equation first formulated by R Hirota: the paper by J A Zagrodziński and T Nikiciuk, in the context of addition formulae for corresponding θ-functions and Abelian functions; the paper by A K Svinin, deriving integrable lattices from the KP hierarchy; the paper by C R Gilson *et al* exhibiting the use of Pfaffian techniques to the discrete KP equation; the paper by X-B Hu and H-W Tam deriving Bäcklund transformations within the Hirota bilinear framework and the paper by S Kakei and Y Ohta deriving new differential–difference systems related to toroidal Lie algebras.

Orthogonal polynomials and special functions. A number of papers highlight the importance of discrete (integrable) equations within the context of special function theory. The paper by S N M Ruijsenaars investigates special coefficient functions arising from analytic difference equations of the Lamé type. M Zygmunt in his paper investigates solutions of difference Schrödinger equations leading to generalized Chebyshev polynomials. The paper by J Harnad *et al* deals with two-dimensional orthogonal polynomials and associated superintegrable systems. A I Aptekarev and W van Assche investigate in their paper the asymptotics of discrete orthogonal polynomials and apply this to the finite Toda lattice. q-special functions of Lauricella's hypergeometric class are shown to lead to solutions of the discrete KP equation in the paper by M Nishizawa. The paper by F A Grünbaum *et al* hits on the issue of bispectral problems, in particular a problem raised by Bochner in 1929, but now answered in the much more general context of matrix problems.

Discrete systems and computation. In this section there are two papers, the one by C Brezinski dealing with discrete dynamical systems coming from sequence transformations, and the contribution by M J Ablowitz *et al* dealing with the effects of discretizations on numerical computations.

Cellular automata, patterns and aggregates. There are five papers under this heading. The paper by G Hatayama *et al* demonstrates the link between representation theory of Lie algebras and automata. The papers by M S Alber and A Kiskowski and by D Takahashi *et al* highlight pattern formation and aggregates, whereas the paper by K Nishinari applies automata to

problems in traffic flow. The paper by S Matsutani, dealing with Lotka–Volterra models in p-adic spaces is grouped here because of the role that other fields (such as with non-Archimedean valuation) may play within a setting of integrable discrete systems.

We are confident that the contibutions to this special issue span to a large extent the breadth of activity in this new field. They demonstrate the amazing richness of the subject area and its potential for future work. Hoping that this issue proves to be useful both for specialists as well as newcomers to the field, we would like to thank all the contributors, as well as others who have assisted in the preparation of this special issue, for their valuable efforts.

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Guest Editors